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The Economics of Spatial-Dynamic Processes

Applications to Renewable Resources

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**The Economics of
Spatial-Dynamic Processes:
Applications to Renewable Resources**

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The Economics of Spatial-Dynamic Processes: Applications to Renewable Resources

“When one tugs at a single thing in nature, he finds it attached to the rest of the world.” John Muir

I. Introduction

A challenging class of environmental/resource problems that has only recently begun to attract the systematic attention of economists emerges when ecological, physical, and economic systems are linked by spatial-dynamic mechanisms. Some examples include: epidemics, invasive species spread, animal disease transmission, subsurface contamination of porous aquifers, shoreline change, forest pest invasion, reserve site selection, provision of ecosystem services, and management of marine and terrestrial species metapopulations. A common feature of these examples is that each depends upon biophysical mechanisms that can be characterized as diffusion processes. From a policy perspective, these problems pose familiar questions about when and how much regulatory effort ought to be utilized to mitigate the problems, but also less familiar questions about **where** control efforts should be applied over spatial landscapes.

New developments in the economics of spatial-dynamic processes are timely, and arguably overdue, for several reasons. First, many of the specific problems that motivate the interest in this general class of problems are increasingly in the public eye. While epidemics and invasions are not new, they are certainly more visible recently and likely to become more prevalent as globalization intensifies. Second, new technological developments in remote sensing, GIS, and enhanced computational capacity are generating new data that are spatially explicit and at very fine spatial resolutions. As these data are examined, scientists are

discovering patterns in the organization of the biosphere and human activities that beg explanation. Third, and related, most of the scientific disciplines from which environmental and resource economics draw have become much more spatially focused over the past two decades. Environmental and resource economics lag behind these supporting disciplines in adopting new paradigms that depict spatial-dynamic processes. Finally, these problems governed by spatial-dynamic processes pose interesting new questions about how to design policies and institutions that account for the manner in which spatial-dynamic processes influence spatial interconnectedness. For example, economic systems characterized by decentralized ownership of space cannot be expected to mitigate spatial-dynamic externalities such as the spread of contaminants in groundwater aquifers or the spread of invasive weeds. New institutions and instruments are thus needed to tackle problems that unfold over landscapes with predictable spatial-dynamics.

This paper will survey recent and ongoing research on the economics of spatial-dynamic processes, summarizing and synthesizing conclusions of the handful of papers that have been completed, and then discuss and illustrate opportunities for additional analysis. Spatial-dynamic problems pose special challenges on multiple fronts, from analytical and technical modeling, to derivation and interpretation of new optimality conditions, to challenging empirical problems, to the synthesis of policy conclusions from analytical work. We outline some of these challenges below. In the next section, we discuss the analytical framework for spatial-dynamic problems, focusing particularly on the importance of diffusion mechanisms. In the third section, we illustrate how bioeconomic behavior can help measure diffusion parameters and hence help understand linked bioeconomic spatial-dynamic systems using a simulated data experiment. The final section discusses new institutional challenges raised by spatial-dynamic processes.

II. Analytical Challenges

The most challenging analytical problems associated with studying spatial-dynamic processes stem fundamentally from the need to integrate the spatial dimension into optimization frameworks. Spatial-dynamic problems have characteristics that are relatively unfamiliar to economic modelers, namely 1) diffusion processes; 2) boundary conditions; and 3) spatial geometry.

A. Diffusion Processes

The heart of spatial-dynamic processes is the diffusion process that depicts the interaction between the object of study and time/space. Real biophysical processes exhibit a range of qualitatively different kinds of diffusion processes.¹ The simplest kind of diffusion process begins by assuming that a population (eg. cells, bacteria, animals, fish, insects) diffuses randomly over space. Consider a one-dimensional representation of space over a line. Suppose that the probability that a unit of the population $N(x,t)$ will move left or right is equal. Then it has been shown that such a diffusion process can be represented by the partial differential equation (PDE):

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial N}{\partial x} = D \frac{\partial^2 N}{\partial x^2} \quad (1)$$

where time is indexed with t , space is indexed with x , and the (constant) diffusion coefficient is D .² This well known PDE describes Fick's Law of diffusion, namely that random diffusion (equal probability of moving right or left) will generate a process whereby the flux of a

¹ The most comprehensive treatments of the mathematics of diffusion are in Murray (2002) and Okubo and Levin (2002).

² We are modeling a diffusion process as continuous over time and space. In some applications it may be more illuminating to model the process in discrete time and space, or continuous in time and discrete in space, or discrete in time and continuous in space (Hastings and Botsford, 2003).

concentration of particles (individuals) at a point will be proportional to the gradient of the concentration.

Suppose now that we have a population that disperses in a way described by (1) and that there is a release of N_0 units at the origin. Then it can be shown that the concentration at some distance x at date t is:

$$N(x,t) = \frac{N_0}{2\sqrt{\pi Dt}} \exp\left\{\frac{-x^2}{4Dt}\right\} \quad (2)$$

Equation (2), which is a closed form solution of (1), describes a process by which the population spreads over space. Notice that units of the population diffuse from high density locations to low density locations at a rate that depends upon the diffusion coefficient D .³

The diffusion process depicted by the partial differential equation in (1) and its solution in (2) is essentially the simplest spatial-dynamic processes that one might imagine. There are several important ways that the Fickian diffusion process has been generalized. One obvious generalization is an increase in dimensionality to depict diffusion in a two-dimensional plane or 3-dimensional space. Another generalization is to make the diffusion coefficient density dependent. For example, many insect populations are known to be attracted (via pheromones) to high concentrations rather than repelled. Let the diffusion coefficient be density dependent so that:

$$D(c) = D_0(N / N_0)^m \quad (3)$$

³ Evolution of coastline shapes are also governed by spatial diffusion, and small perturbations can propagate over large spatial scales (Ashton, Murray, and Arnoult 2001). Most management of beaches as currently practiced, however, does not consider spatial externalities beyond the immediate surrounding area.

where D_0 and N_0 are constants and $m > 0$. Consider the 2-dimensional diffusion process with source population at the origin and concentration measured at some radius r from the origin.

Then the partial differential equation describing the spread is:

$$\frac{\partial N}{\partial t} = \left(\frac{D_0}{r} \right) \frac{\partial}{\partial r} \left[r \left(\frac{N}{N_0} \right)^m \frac{\partial N}{\partial r} \right] \quad (4)$$

The solution to this PDE has a cross-sectional profile, where the concentration is more bunched near the original release point and there is no tail on the leading edge.

Another generalization involves combining the diffusion process that describes how something spreads over space with a differential equation description of a growth process that occurs at each point. The most famous is the so-called Fisher reaction-diffusion equation:

$$\frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} + \alpha N(1 - N) \quad (5)$$

first proposed by R.A. Fisher in 1937. This spatial-dynamic equation describes a population as being influenced by random diffusion (the first part of the RHS) and density dependent growth at each point in space (the second part). This equation is impossible to solve in closed form, but the solution has been shown to exhibit a traveling wave characteristic, in that as time gets large, the population moves as a wave front with a constant velocity $v = 2\sqrt{\alpha D}$. The speed of the wavefront thus depends upon the product of the diffusion coefficient and the intrinsic growth rate at a population level close to zero.

Finally, many processes of diffusion are influenced by physical forcing that influences the direction of movement in some biophysical medium. For example, larvae in a marine

environment may be influenced by dominant currents, so that whether an organism moves up or down-current is asymmetric. This is called advection. Suppose there is a small bias so that the probability of a particle moving right is larger than the probability of moving left. Then the Fickian diffusion equation becomes:

$$\frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} - V \frac{\partial N}{\partial x} \quad (6)$$

the solution of which exhibits an asymmetric spatial diffusion pattern relative to the origin (Petrovskii and Li, 2003).

B. Boundaries and Spatial Geometry

While the heart of a model of spatial-dynamic processes is the partial differential diffusion relationship describing state variables as a function of time and space, other important components are the geometry of space and the boundary conditions. In the real world, we see great diversity in both geometry and boundaries. The famous invasive species case of the gypsy moth has been progressing at a relatively predictable pace as a wave front for the past century, radiating out from its known point of introduction in 1869 near Boston, Massachusetts, and moving at a rate of about 20 kilometers per year (Liebhold et. al., 1992). In this case, the geometry may be thought of as a two-dimensional plane delineated by a subset of the North American continent. While the actual physical boundaries that limit the spread of the gypsy moth may be the whole of North America, the gypsy moth will bump up against a fixed barrier when it reaches the Western edge of the continent. Boundary conditions may be relevant because they influence the qualitative nature of the diffusion process, or they may be relevant because a policy setting suggests a boundary. For example, a government may view the relevant boundaries over which it wishes to control an invasive species as its political boundaries, even though the pest might actually move through such boundaries to neighboring political entities.

As another example, plumes of subsurface contaminants spread through soils and into aquifers in ways well predicted by Fickian diffusion, calibrated by diffusion relationships that reflect the porosity and conductivity of soil. But the geometry and boundaries of particular cases may be fixed by aquifer edges, cap rocks, and other impermeable media. Similarly, every year flu epidemics spread from Asia to North America, entering at ports on the West Coast like Seattle and San Francisco, jumping to other airline hubs in Denver, Chicago and New York, and radiating out from those cities (Viboud et. al. 2006). From a modeling perspective, some processes are sensibly modeled over an infinite one- or two-dimensional space which absorbs an asymptotically disappearing concentration of particles. For other processes, the actual physical boundary is important simply because it constrains the damages.

Modelers generally depict boundaries as being either absorbing, reflecting, or some hybrid characterized in terms of gradients. Absorbing boundaries imply that the state variable is zero at the boundary (appropriate at a continental edge). Reflecting boundaries bounce the diffusing particles back into the space (an enclosed aquifer) and zero flux boundaries depict a particle as orthogonal or at some other angle at the boundary. Finally, spatial geometry is important and dependent upon the mosaic that contains the spatial-dynamic process. It matters whether the medium is homogeneous, or whether it contains spatial character such as bottlenecks, edges, hostile patches, and heterogeneous productivity.

C. Bioeconomic Models with Spatial-dynamic Processes

Aside from characterizing the diffusion, boundary and spatial geometry of spatial-dynamic processes, the other analytical challenge relates to: 1) linking biological and physical system descriptions to economic system descriptions; and then 2) describing policy consequences, particularly under different institutional settings. We have models of open access,

regulated open access, regulated restricted access, and sole-owner optimization that can be adapted to spatial-dynamic settings. Consider the case of the first-best optimal exploitation plan for a resource. Resource economists are familiar with applications of dynamic optimization theory to conventional resource use problems that express the dynamics of resource stocks as ordinary differential equations (ODEs). The mathematics of these problems yields well-known solutions that are also systems of ordinary differential equations in the stock equations and the control or co-state equations.

To illustrate how optimization of a system characterized by spatial-dynamic processes differs from conventional results, consider a renewable resource characterized by a Fisher reaction diffusion equation on a one-dimensional line. The population would then evolve over space and time in a manner determined by the harvest rate at each point, which we might assume to depend upon effort $E(x,t)$ and biomass according to:

$$\frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} + \alpha N(1 - N) - q(x,t)E(x,t)N(x,t) \quad (7)$$

Now, suppose that we define the relevant space over which this resource can survive in terms of some boundaries $[x_l, x_u]$. Then the optimization problem can be written as:

$$\max \int_{x_l}^{x_u} \int_0^{\infty} \exp(-\rho t) \{B[E(x,t), N(x,t)]\} dt dx \quad (8)$$

subject to:

$$\frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} + \alpha N(1 - N) - q(x,t)E(x,t)N(x,t) \equiv D \frac{\partial^2 N}{\partial x^2} + g\{E(x,t), N(x,t)\}$$

where the objective is to maximize some concave benefit function subject to the PDE state equation and boundaries. This problem has recently been characterized by Brock and

Xepapadeas (2006), who show that the (interior) solution can be expressed in terms of a simultaneous system of two PDEs, namely:

$$\frac{\partial \lambda}{\partial t} = \rho \lambda(x,t) - \frac{\partial H}{\partial N} - D \frac{\partial^2 \lambda}{\partial N^2} = \rho \lambda(x,t) - \{B_N + \lambda(x,t)g_N + D \frac{\partial^2 \lambda}{\partial N^2}\} \quad (9)$$

$$\frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} + g\{E^*(x,t), N(x,t)\}$$

where $E^*(x,t)$ is derived by maximizing the Hamiltonian at every point in space and time. The important conclusion is that when the underlying system is a spatial-dynamic system, the fundamental state and co-state equations of the optimized Hamiltonian are diffusion equations, expressed as partial differential equations (PDEs) in both time and space. The solutions to spatial-dynamic problems thus are derived from systems of PDEs that are notoriously more difficult to solve and characterize than ODEs. Because of this, there are only a handful of papers in the mathematics literature that address the optimization of systems characterized by PDEs.

Some qualitative properties can be deduced simply by examining the above system however. Note that the combined system is “self-adjoint” in the sense that both PDEs are diffusion equations with a common diffusion coefficient appearing with negative and positive signs. Consider a steady state for the above system where the right hand sides are zero and there is some optimal spatial equilibrium distribution of biomass. The self-adjoint property suggests that in areas where the biomass variable is generating flux (diffusion flowing from high biomass to low biomass areas) across a particular stretch of space, the corresponding co-state variables will be generating flux in the opposite direction (eg. from low shadow values to high shadow values). This mirrors a property of non-spatial systems, namely that low biomass levels imply high shadow values but it also suggests that the shadow value of another unit of biomass may differ depending upon where the biomass is located in space.

Aside from the structure that emerges from the modified spatial Pontryagin conditions, the other important component of spatial-dynamic systems that drives the nature of the solution is the set of assumptions that are made about boundary conditions. Boundary conditions generally characterize how the system’s initial conditions are embedded, and they also characterize conditions at the boundary of the spatial domain and at the terminal period. Brock and Xepapadeas conclude that the spatial-dynamic analogue of the intertemporal transversality condition for infinite horizon problems can be written as:

$$\lim_{T \rightarrow \infty} e^{-\rho T} \int_{x_l}^{x_u} \lambda(x, T) N(x, T) dx = 0 \quad (10)$$

In addition, they derive spatial transversality conditions that mirror the assumptions that characterize the spatial boundaries. For example, one assumption that might be made is that of a “hostile” boundary. A hostile boundary requires that the population dies if it crosses into the hostile domain. A terrestrial species landing on an island would face a hostile boundary at the island’s edge. Brock and Xepapadeas show that this situation is equivalent to the non-spatial case with a fixed endpoint. Another assumption that might be made is that there is zero flux across a spatial boundary. This is sometimes also referred to as a reflecting boundary. Brock and Xepapadeas show that this case is equivalent to requiring a solution to terminate on a spatial manifold. Finally, if one wanted to eliminate the role of boundary conditions, it might be assumed that space is configured in a circle, so that the end point and beginning points are equal and the state and co-state variables do not jump at the boundary.⁴ Each of these three boundary assumptions and their corresponding transversality conditions are:

Hostile boundary: $N(z_l, t) = 0 = N(z_u, t) \Rightarrow \lambda(z_l, t) = 0 = \lambda(z_u, t)$

⁴ In the new economics of geography, the circle setting is consistent with the geometry of the “racetrack” economy (Fujita, Drugman, and Venables 1999).

Zero flux:
$$\left. \frac{\partial N(z,t)}{\partial z} \right|_{z=z_l} = 0 = \left. \frac{\partial N(z,t)}{\partial z} \right|_{z=z_u} \Rightarrow \frac{\partial \lambda(z_l,t)}{\partial z} = 0 = \frac{\partial \lambda(z_u,t)}{\partial z} \quad (11)$$

Circle:
$$N(z_l,t) = N(z_u,t) \text{ and } \frac{\partial N(z_l,t)}{\partial z} = \frac{\partial N(z_u,t)}{\partial z} \Rightarrow$$

$$\lambda(z_l,t) = \lambda(z_u,t) \text{ and } \frac{\partial \lambda(z_l,t)}{\partial z} = \frac{\partial \lambda(z_u,t)}{\partial z}$$

Any of these may be appropriate in a particular setting, depending upon the nature of the geometry and other features of the setting. The important point is that there are spatial transversality conditions that are analogous to more familiar conditions for non-spatial intertemporal optimization problems. These spatial transversality conditions can be chosen to fit the particular geometry and boundary conditions appropriate to the problem at hand, and they will influence the nature of the optimal solution accordingly.

D. Some preliminary findings

At present, a handful of papers have begun to explore spatial-dynamic optimization problems for special cases and intriguing (and sometimes contradictory) results are beginning to emerge. For example, a paper by Neubert (2003) applies optimal control theory to a fisheries problem depicted on a line, where growth at each point is also affected by spatial diffusion (the Fisher reaction-diffusion model). Neubert reaches the conclusion that it is optimal to maximize exploitation near the boundary, a result that clearly is driven by the hostile boundary assumption that fish die as they reach to border. Interestingly, Neubert also finds that, as one moves inward to the interior of the space, it is optimal to leave certain areas unfished. With a large space over which to optimize, the solution involves a ‘spatial bang-bang’ character, with maximum exploitation alternating with zero exploitation, and a ‘spatial chattering’ pattern near the center with on and off harvesting alternating in infinitely diminishing space. All of these results emerge

when the objective is to maximize physical yield; spatial chattering is not optimal when the objective is to maximize total economic rather than physical returns.

Sanchirico and Wilen (2005) use a continuous time, discrete space model to examine similar questions but under a broader range of circumstances.⁵ They find that spatial heterogeneity is a critical determinant of qualitative conclusions. In a renewable resource metapopulation system⁶ with homogeneous bioeconomic parameters and simple symmetric and homogeneous diffusion, the optimal solution for the system will be identical to the solution that optimizes rents in each patch. On the other hand, with bioeconomic heterogeneities, S/W find that total effort, the spatial distribution of effort, and the gains from rationalization depend importantly on the spatial distribution of these heterogeneities as well as the structure of the diffusion system. A particularly intriguing finding is that many conclusions depend upon whether biological and economic gradients are aligned or opposing. For example, in a system in which biomass flows from north to south, qualitative conclusions differ depending upon whether unit profits also line up from north to south, or vice-versa, or in some other ordering.

Costello and Polasky (2006) develop a stochastic metapopulation model and, in some cases, come to more or less the opposite of the S/W conclusions. In particular, they demonstrate some (special) conditions under which optimizing a linked stochastic metapopulation can be undertaken on a patch-specific basis without considering diffusion and linkages across patches. Lastly, Brock and Xepapadeas apply optimal control theory to a continuous in time and space

⁵ The basis of the Sanchirico and Wilen model is the Clark (1990) two-patch model of optimal spatial exploitation. Using a similar formulation, Tuck and Possingham (1994) and Supriatna and Possingham (1992) investigate optimal management in a two-patch sink-source system with no economic heterogeneity, and Huffaker et. al. (1992), Bhat et. al. (1993, 1996), Lenhart and Bhat (1999), Skonhofs and Solstad (1996), and Schulze and Skonhofs (1996) investigate optimal management in a two-patch setting for terrestrial species. Janmatt [11] develops a patchy model and investigates the optimal spatial management for a northeast Atlantic clam fishery. Brown and Roughgarden (1999) use an optimal control model with sedentary adults and larval dispersal to illustrate the value of larval pools to system wide fishery profits.

⁶ A metapopulation is a system consisting of local populations occupying discrete habitat patches with significant demographic connectivity between patches (Sale et al. 2006).

renewable resource model, and generate completely different conclusions. They begin with a model that is homogeneous with respect to bioeconomic parameters, with simple Fisher reaction diffusion state dynamics. They optimize a system assumed to lie on a circle (so that boundaries do not affect the results) over an infinite horizon. They examine the stability of the solution and postulate that it is possible under certain parameter combinations to have an equilibrium with heterogeneity in optimal effort, biomass and harvest over space, even in a system that is parametrically homogeneous over space.

These conclusions are the first to emerge from this new frontier area examining the economics of spatial-dynamic processes and they are provocative and considerably more complex than conventional non-spatial results. These papers by Neubert, Sanchirico and Wilen, Brock and Xepapadeas, Costello and Polasky, and others tackle only one class of problems, namely those dealing with renewable resource systems.⁷ Nevertheless, the results are provocative and we will no doubt accumulate more results from additional cases. The task will then be to synthesize and generalize conclusions from modeling exercises and develop an intuition about these problems. There are equally compelling problems that we might refer to as point-input problems, where a spatial-dynamic process is initiated as the result of a propagule at a point in space/time. Invasive species problems are good examples, and these are more akin to non-renewable resource problems, where uninvaded space is “used up” over time, depending upon how and where control efforts are applied. There is much less work done to date on these problems, particularly where space is explicit (Sharov and Liebhold 1998, Potapov et. al.2006).

III. Empirical Challenges and Opportunities

⁷ Some recent work by other economists on the spatial implications of resource use include: Albers (1996), Bulte and Cornelis (1999), Wilson et al. (1999), Robinson et al. (2002), and Holland and Schnier(2006). For a general discussion of the role of spatial heterogeneity in economics, see Bockstael (1996).

We make the argument in Section II that a critical component, if not the most important component, of spatial-dynamic systems is the diffusion mechanism linking space and time. Scientists know something about diffusion processes for some systems that are either easily observed and measured or those that can be measured from experiments. For other systems, however, understanding the critical diffusion relationships is problematic. Diffusion of liquids (water, oil, contaminants) in permeable soils is difficult to understand because of the heterogeneities in subsurface soil structure as well as inclusions and barriers that are difficult to observe. Similarly, populations of adults and juveniles moving in time and space in marine systems are particularly difficult to measure because they often cannot be directly observed. But the manner in which spatial-dynamics are determined jointly from both biophysical and economic processes raises a provocative question; namely can bioeconomic observations be exploited in order to infer and measure unobserved diffusion coefficients? In other words, can we observe human users of spatial-dynamic marine system and infer anything about the underlying biological structure generating their behavior?

The logic behind these questions is as follows. First, some spatial-dynamic systems with diffusion will tend without exploitation to reach an equilibrium in which zero net dispersal maintains stasis in various patches of a metapopulation, e.g. density-dependent systems. In others, spatial equilibrium will involve net dispersal, e.g. sink-source systems. In either case, exploitation by humans may perturb metapopulation dynamics in ways that generate useful variation in biomass levels and diffusion (Sanchirico and Wilen 1999; Sanchirico 2005). These perturbations may reveal system characteristics in ways that are measurable by observing the behavior of fishermen on the surface of the system. In this section, we show with a Monte Carlo exercise how it is possible to observe fishermen behavior and performance in ways that

illuminate and allow the statistical recovery of parameters that govern the underlying structure of the metapopulation system they are exploiting.

There is a long history of estimating biological parameters from economic data in a fishery that dates back to Schaefer (1954).⁸ The essence of the problem is that fish stocks are unobserved, but catches and fishing effort are observed. These quantities from the fishery are linked to fish stocks through a production function, and this linkage allows one to substitute observables recursively into the biological state equation and estimate parameters. Here, we first show that this approach can be used to estimate bio-physical parameters of spatial-dynamic processes using a discrete-time version of the Sanchirico and Wilen (1999) model.

A. Modeling a sink-source system with fishing micro-data

As a starting place, we assume a sink-source dispersal system. The sink-source system captures the potential for oceanographic features such as currents and bathymetry to influence the spatial diffusion process by influencing flows of larvae and/or adults. Let j index own patch in a system with J total patches. The state equations are then of the following form:

$$X_{jt+1} = X_{jt} + r_j X_{jt} \left(1 - \frac{X_{jt}}{K_j} \right) - d_{jj} X_{jt} + \sum_{k \neq j} d_{jk} X_{kt} - H_{jt} + \varepsilon_{jt} \quad (12)$$

In this model, H , r , and K are patch-specific harvest, intrinsic growth, and carrying capacity respectively. The d 's are dispersal coefficients, and ε is known in the ecology literature as process error. We use a Schaefer harvest function to link the economics:

$$H_{jt} = q X_{jt} E_{jt}^{TOT}, \quad (13)$$

⁸ A few noteworthy examples in economics are Comitini and Huang (1967), Wilen (1976), and Deacon (1989).

where $E_{jt}^{TOT} = \sum_{i=1}^n E_{ijt}$ is the total effort in patch j at time t adding up across all n fishery

participants. Defining the aggregate patch catch-per-unit effort as z , we can rewrite the state equation as:

$$\frac{z_{jt+1}}{q} = \frac{(1+r_j-d_{jj})}{q} z_{jt} + \left(\frac{-r_j}{q^2 K_j} \right) (z_{jt})^2 + \sum_{k \neq j} d_{jk} \frac{z_{kt}}{q} - H_{jt} + \varepsilon_{jt}. \quad (14)$$

Note that we are assuming no observation error, i.e. the harvest functions hold without error.⁹

Multiplying both sides by q , we have a linear estimating equation for each patch (with a total of J patches) that is solely in terms of observables:

$$z_{jt+1} = \sum_{k=1}^J \beta_{jk} z_{kt} + \beta_{jJ+1} (z_{jt})^2 - \beta_{jJ+2} H_{jt} + \varepsilon_{jt}^* \quad (15)$$

Two notes are in order. First, the number of parameters in each equation is growing with the number of patches, and the number of equations is also growing with the number of patches. Together, the total number of parameters is growing by the square of the number of patches. In a similar manner, a spatial weighting matrix in spatial econometrics grows with the square of spatial interconnections, increasing the computational burden. But in our case, the issue is identification and not necessarily computation. Second, the diagonal elements of the dispersal matrix are not separately identified from intrinsic growth. This poses no problem in a sink-source system as long as spatial adding up exists, $d_{jj} = \sum_{k \neq j} d_{kj}$. In words, the outflow from patch j must exactly equal the inflow from patch j to all other patches.

⁹ In general, observation error that is not modeled in empirical settings will lead to errors-in-variables bias in the biological parameter estimates (Uhler 1980). We assume initially no observation error to obtain a tractable starting point in our simulations. Ecologists are continuing to develop models that incorporate both process error and observation error (De Valpine and Hastings 2002). Recent work by economists in a non-spatial system exploits the panel structure of fishery data to address simultaneously process error, observation error, and the limitations of the Schaefer production function (Zhang and Smith 2007).

One could proceed to generate simulated data from the biological state equations and generate random economic data as a Monte Carlo design. However, we are interested in querying the system to find circumstances—both bio-physical and economic—that are likely to generate data that will resolve the true structure. Thus, we choose a behavioral model structure that is consistent with empirical studies of fishing behavior over space and time (Eales and Wilen 1986; Dupont 1993; Holland and Sutinen 2000; Smith 2005). To that end, we use a discrete choice framework to generate the economic data. Suppose that there are n permitted fishing vessels in a limited entry fishery, which we will assume are owner-operated. We index the vessels by i . For simplicity, we match the time scale of fishing choice occasions to the time scale of the biological dynamics, so t indexes the choice occasion.¹⁰ Denote fishing site as j as in the biological model, and following Smith (2002), we also model the alternative of not fishing ($j = 0$). The utility of each choice (U) can be broken into a deterministic and random component:

$$U_{ijt} = v_{ijt} + \eta_{ijt} \quad (16)$$

For simplicity, as a starting point we will assume that the η_{ijt} 's are i.i.d. Type I Extreme Value.

The deterministic portion of utility from not fishing is value of some outside opportunity (α), which could reflect the value of leisure or wages from another employment activity. The deterministic portion of a fishing alternative is the profitability of fishing, which includes revenues from fishing, a fixed cost of taking a trip (bait, ice, etc.) denoted by c , and travel cost (ϕl):

$$v_{ijt} = \begin{cases} \alpha, & \text{for } j = 0 \\ p_t h_{ijt} - c - \phi l_{ij}, & \text{for } j = 1, 2, 3, \dots, J \end{cases} \quad (17)$$

¹⁰ In general, the relative time scale over which the biological and economic processes unfold over space is an important factor in determining the underlying spatial-dynamic processes.

where h is individual expected harvest, p is price of fish, and l_{ij} is the pairwise length of roundtrip travel distance from individual i 's port to patch j . These distances introduce a source of spatial variation in the economic gradient that is potentially independent of spatial variation in the bio-physical gradient. Outside opportunities (α) could also vary systematically over space and time in ways that are uncorrelated with spatial-dynamic processes in the water, but we assume this away initially. Our assumption of no observation error effectively means that actual harvest is expected harvest, but it is important to note that in empirical settings the two are not equivalent. Given our assumption of the Schaefer production function above and noting that E takes on a value of 1 if the site is chosen and 0 otherwise, we can substitute to form the random utility:

$$U_{ijt} = \begin{cases} \alpha + \eta_{ijt}, & \text{for } j = 0 \\ p_t q X_{jt} - c - \phi l_{ij} + \eta_{ijt}, & \text{for } j = 1, 2, 3, \dots, J \end{cases} \quad (18)$$

Here we see that the spatial bio-physical gradient enters through the patch-specific stocks.

In the static random utility maximization framework (RUM), the individual is assumed to select the choice with the highest utility for each choice occasion. The analyst observes components of deterministic utility but not the idiosyncratic error term. As a statistical problem, with data on trips to each location, prices (varying over time), stocks (varying over time and space), and travel distance (varying over space), one could estimate three parameters from this model: q , ϕ , and $(-c-\alpha)$. In McFadden's (1974) conditional logit model, these parameters are only estimated up to scale. Note also that only the sum of the outside opportunity and the trip fixed cost can be identified because there is only one constant in the utility differences. In what follows, we use the RUM framework combined with the spatial-dynamic state equations to generate simulated data, then recover bio-physical parameters in a first stage using aggregate

data, and recover economic parameters in a second stage using disaggregate data. Because catchability is identified in the first stage, we can actually recover the scale coefficient in a second stage, allowing us to give an explicit structural interpretation to each parameter.

We use the following algorithm to simulate data from the bioeconomic model. First, we determine the number of individuals (n), choices ($J+1$), and choice occasions (T); assign true values to all of the bioeconomic parameters including the mean and standard deviation of price; draw a vector of prices; and set initial conditions on the stocks (X_{0j}). Second, we form the deterministic portion of utility at $t=0$ in (18) for each individual in each location. Third, we draw i.i.d. Type I Extreme Value errors η_{ijt} by inverting a uniform distribution ($u \sim U[0,1]$) using the cdf of Type I (denoted $F(\eta)$) to recover the errors that we need: $\eta = b \left\{ -\ln \left[-\ln(u) \right] \right\}$, where b controls the variance of the distribution).¹¹ Fourth, we fill in the random component of utility in (18) and simulate individual choices:

$$E_{ijt} = \begin{cases} 1, & \text{if } U_{ijt} = \max(U_{i0t}, U_{i1t}, U_{i2t}, \dots, U_{iJt}) \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

Fifth, we add up individual-, space-, and time-specific effort in (19) and substitute into (13) to obtain total harvest by patch. Sixth, total harvest is substituted into (12). Finally, we draw an i.i.d. normally distributed process error in (12) and iterate the state equation. We return to step two and follow the process for $t=1$, and so forth.

To recover the bio-physical spatial-dynamic parameters from the simulated data set, we first estimate the reduced-form parameters in (15). Note that only aggregated catches and catch-per-unit-efforts enter the estimation equations, so in the first stage we add up across all of the individuals. Because the model is linear in all of the observables, estimation can be done using

¹¹ See Train (2003) for details on the inversion method for drawing from distributions.

Seemingly Unrelated Regression (SUR) and imposing the cross-equation restriction

$\beta_{j,j+2} = \beta_{k,j+2}, \forall j, k$ (to account for the assumption that catchability is the same over space). The next step is to transform the reduced-form parameters into the structural parameters using (14) and (15).

Table 1 contains a summary of 100 simulated data experiments. We assume a 3-patch sink-source system ($J=3$) with 100 periods ($T=100$) and 100 individuals ($n=100$). Individuals are distributed randomly over space with mean travel distances of 3, 2, and 1 for patches 1, 2, and 3. Appendix A provides details on recovering the structural parameters of the system. The top set of results in Table 1 reports summary statistics of the recovered structural parameters assuming that we know the true connectivity structure of the spatial system. That is, we know which elements of the full dispersal matrix are zero. In this case, the form of the sink-source system imposes four restrictions. There are thus nine free parameters to estimate (compared to 13 in a system where each patch is connected to every other patch). For all parameters, the mean and median are close to the true value. Even the minima and maxima are qualitatively correct for all parameters except d_{21} and d_{31} , for which the minima are negative. As in any Monte Carlo experiment with an unbiased estimator, the spread of the resulting parameter estimates is a function of how much noise the analyst injects into the system. Thus, the top half of Table 1 primarily serves to illustrate that, in principle, one could recover all of the bio-physical parameters of a spatial-dynamic system from economic data alone. To our knowledge, this type of analysis has never been attempted empirically, though the availability and reliability of spatially-explicit fishing logbook data makes the exercise possible.¹²

¹² It is important to note that the mandatory use of electronic logbooks is increasing throughout the world's fisheries, which makes one even more optimistic that such exercises will be more feasible in the future.

The bottom half of Table 1 demonstrates that in some circumstances, not knowing the structure of the dispersal matrix still permits the recovery of the structural parameters. The structural coefficients do not appear to be biased in spite of including regressors that should not be included. This is a reflection of the underlying structure of our data generating process and the properties of SUR; inclusion of additional regressors should only affect efficiency. We can see this efficiency loss simply by comparing the standard deviations across sets of coefficients in the top and bottom half. There is more spread in the bottom half. Appendix B reports results for increased process error and a different set of sink-source parameters. Qualitatively, the same conclusions hold.

In principle, one could test down from the unrestricted model to the true model. Suppose that we know which joint restriction to test. In each 100 cases summarized in Table 1, we perform a likelihood ratio test and find that we fail to reject the restriction 98 times. That is, when we know the proper restriction to impose, we only incorrectly reject it 2 times out of 100. Nevertheless, it is important to consider that real world data will not necessarily conform to the data generating process of our simulated data environment. Our ability to understand bio-physical spatial-dynamic processes may require that metapopulation ecologists or ocean circulation modelers specify the qualitative structure of spatial dispersal. Quantitatively, we may be able to recover the parameters from economic data alone, but the efficiency of these estimates is hampered by spending degrees of freedom on incidental spatial parameters. This problem is likely worsened by the multicollinearity of stocks over space. With the expansion of remote sensing, GIS, GPS, and fish tagging technologies, the provision of qualitative information is not an unrealistic possibility. If scientists are able to tell us that the probability of something flowing

from A to B is very low but the probability of something flowing from B to C is high¹³, economic data may be able to tease out how much.

In the absence of qualitative information about spatial structure, our simulated data experiment raises question about the circumstances under which economic data can reveal the true structure under various spatial configurations. Given our data generating process, when we know the set of restrictions to test, we are able to test down accordingly. A natural question to ask: is whether there is a logical order in which to test down from the general dispersal matrix when we do not know the restriction to impose. Since we have established that the economic data can, in principle, allow the recovery of spatial parameters, designing an econometric approach that will distinguish different types of spatial structure becomes an important question for future research. This endeavor will be complicated by empirical realities of matching spatial scales. Fishing grounds, as reported in logbooks, are not necessarily the same as patches in a metapopulation. Moreover, the stacked-equation approach that we develop above implicitly assumes a discrete rather than continuous space. Matching the discreteness of spatial locations in logbooks with a continuous-space model will provide a different set of challenges.

Setting aside the qualitative spatial structure, our simulated data experiments provide quantitatively reasonable estimates of the true parameters because we build in sufficient variation in the data generating process. When will this be true for real-world data sets? Many systems are either limited entry (like our simulated data experiment), open access, or regulated open access. In all cases, participants over the long run have an incentive to dissipate rents over space (Sanchirico 2004). For our particular setting, this generates spatial correlation in stocks. S/W (2005) analytically show that the alignment or opposition of spatial biological and economic

¹³ See, for example, the recent work on connectivity structures by Cowen et al (2000) and Cowen, Paris, and Srinivasan (2006).

gradients affects the qualitative character of a spatially optimized system. Biological and economic gradients may combine in a manner that generates large or small amounts of variation in economic data. One hypothesis worth examining is that offsetting biological and economic gradients will generate more variation and make it easier to recover spatial parameters from economic data.¹⁴

Though there are further issues to address in the recovery of bio-physical spatial-dynamic parameters, we can use a second stage to recover economic parameters. Continuing with the assumption that we do not observe stocks directly, we use catch-per-unit effort as a proxy in the second stage. Because $X_{jt} = \frac{z_{jt}}{q}$, q cancels in (18) and the coefficient on $p_t z_{jt}$ is one. The simplicity of the stock index is a result of the Schaefer production function, but the result is general; we can substitute out the stock in individual expected harvest with an aggregate measure because individuals fishing in the same place at the same time are facing the same stock. By estimating the conditional logit model, we thus recover the otherwise unidentified scale coefficient. Note that whether we know the true structure of the bio-physical system does not affect the second stage estimation; the model and data are the same. Table 2 contains results of 100 runs to recover structural economic parameters. The true scale coefficient is 2.0, so not surprisingly we see that the raw estimates are approximately one half their true values. After using the revenue coefficient to transform the remaining coefficients, the recovered structural parameters are close to the true values. These results, combined with Table 1, illustrate a complex sorting equilibrium that emerges from the interaction of human agents with a spatially heterogeneous natural system (Phaneuf, Herriges and Carbone 2007). Our ability to measure all

¹⁴ A possible source of an offsetting economic gradient is the location of fishing ports relative heterogeneous subpopulations. However, the intensity of effort fishing from a particular port in real systems can be endogenously determined by the overall spatial productivity of the resource (Smith and Wilen 2004)

parameters of interest is not due to the system being in a spatial-dynamic steady state; instead, we are able to exploit a sequence of non-price equilibria that are imposed bio-physically by virtue of the state equations holding in each period and our knowledge of a bioeconomic link through the production function.

B. Modeling a density-dependent system

In contrast to a sink-source system, a density-dependent system is expected to characterize spatial diffusion of adults due to ecological effects such as crowding and competition for food resources (Sanchirico and Wilen 1999). If the population in patch 1 relative to patch 1 carrying capacity is higher than the population in patch 2 relative to patch 2 carrying capacity, then we expect flow from patch 1 to patch 2. Density-dependence imposes a particular structure on the dispersal parameters in equation (12). For the three-patch system, we have:

$$X_{1t+1} = X_{1t} + r_1 X_{1t} \left(1 - \frac{X_{1t}}{K_1}\right) + a_1 \left(\frac{X_2}{K_2} - \frac{X_1}{K_1}\right) + a_2 \left(\frac{X_3}{K_3} - \frac{X_1}{K_1}\right) - H_{1t} + \varepsilon_{1t} \quad (20)$$

$$X_{2t+1} = X_{2t} + r_2 X_{2t} \left(1 - \frac{X_{2t}}{K_2}\right) + a_1 \left(\frac{X_1}{K_1} - \frac{X_2}{K_2}\right) + a_3 \left(\frac{X_3}{K_3} - \frac{X_2}{K_2}\right) - H_{2t} + \varepsilon_{2t} \quad (21)$$

$$X_{3t+1} = X_{3t} + r_3 X_{3t} \left(1 - \frac{X_{3t}}{K_3}\right) + a_2 \left(\frac{X_1}{K_1} - \frac{X_3}{K_3}\right) + a_3 \left(\frac{X_2}{K_2} - \frac{X_3}{K_3}\right) - H_{3t} + \varepsilon_{3t} \quad (22)$$

Rearranging, equations (20) through (22) can be expressed as equation (12) with the following dispersal matrix D :

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} = \begin{bmatrix} -\left(\frac{a_1 + a_2}{K_1}\right) & \left(\frac{a_1}{K_2}\right) & \left(\frac{a_2}{K_3}\right) \\ \left(\frac{a_1}{K_1}\right) & -\left(\frac{a_1 + a_3}{K_2}\right) & \left(\frac{a_3}{K_3}\right) \\ \left(\frac{a_2}{K_1}\right) & \left(\frac{a_3}{K_3}\right) & -\left(\frac{a_2 + a_3}{K_3}\right) \end{bmatrix}. \quad (23)$$

As in the sink-source system, there are three adding up restrictions. However, the complication of estimating this model with SUR is that the model is overidentified, and restrictions on the model to exactly identify it involve nonlinear implicit functions.

Table 3 reports results from the SUR estimates based on 100 Monte Carlo data sets generated by a density-dependent system. In spite of overidentification, the means and medians of the parameter estimates track the true parameters. The true value of a_1 is 0.25. Using d_{21} and K_1 to recover a_1 , we find 0.23019, whereas using d_{12} and K_2 , we find 0.2569. Although these values differ, for purposes of optimizing the spatial-dynamic bioeconomic system (as in Sanchirico and Wilen 2005), we need the dispersal matrix itself and not necessarily the underlying structural parameters of it (a_1 , a_2 , and a_3). Thus, we argue that our numerical experiments provide some reason for optimism in using economic data to estimate bio-physical parameters of a spatial-dynamic system. Nevertheless, as the number of patches grows, the number of structural parameters in a density-dependent system also grows. Whether an estimated overidentified dispersal matrix diverges more or less from the true dispersal matrix requires further exploration.

C. Incorporating observation error

The empirical population dynamics literature has long recognized that process error, i.e. stochasticity in stock-recruitment relationships, is just one complication; models that construct observable indices for the unobservable stock must also account for observation error. Here we conduct experiments with the density-dependent system from above that introduce observation error on top of the process error.

In the context of using economic data from a fishery to recover stock dynamics, the observation error problem amounts to the production function, i.e. our equation (13), involving

an error term. When the catch-per-unit effort is then substituted into the state equation, the resulting estimation problem will suffer from errors-in-variables bias. This problem was diagnosed in the fisheries literature by an economist (Uhler 1980) and has since drawn considerable intellectual attention from biologists, fisheries managers, and economists. To add observation error, we rewrite equation (13) with an individual-level error as:

$$H_{jt} = qX_{jt} \sum_{i=1}^n E_{ijt} \xi_{ijt} \quad (24)$$

where $\ln \xi_{ijt} \sim N(0, \sigma_{\xi}^2)$. While still a simple production function, this error structure is consistent with much of the fisheries production literature that estimates harvest functions in logarithms.¹⁵ Because we know that intrinsic growth and carrying capacity in a single-equation model will be biased, we explore the spatial system with a larger number of simulations in order to see how significant these biases are.

Table 4 reports simulations of the density-dependent system with 5,000 Monte Carlo data sets for three different levels of observation error σ_{ξ} : 0.1, 1.0, and 2.0. Not surprisingly, biases are small with low observation error. This is consistent with the literature that empirically estimates single-equation stock dynamics. As observation error increases, bias grows but in a way that is different across types of parameters. In particular, the traditional logistic growth parameters and catchability become nonsensical as observation error becomes very large ($\sigma_{\xi} = 2.0$). In these cases, median intrinsic growths and catchability are negative, and median carrying capacities are more than three times their true values. These results echo a principal concern of population biologists, namely that observation error can render the recovered

¹⁵ Note that everyone fishing at the same time in the same place is facing the same stock. This is the sense in which disaggregated data can be used to estimate production functions, but it is also the key point that allows Zhang and Smith (2007) to difference out the stock and use panel methods to account for observation and process error. This method has not yet been extended to the spatial domain.

population parameters misleading for management purposes (Polacheck, Hilborn, and Punt 1993). However, the biases in the dispersal matrix remain relatively small even in the presence of huge observation error. Qualitatively, the means and medians all have the correct signs, and all of the dispersal parameter medians are within a factor of two of the true values. This suggests that economic data, even without accounting for observation error, can be helpful in understanding spatial-dynamic bio-physical processes. Although this result is surprising, our numerical experiments do not establish its generality, warranting further research.

IV. Institutional and Policy Design Issues

In the final analysis, the purpose of modeling spatial-dynamic processes and subjecting these to various optimization techniques is not only to understand the nature of the human/ecosystem interactions, but also to suggest policies that might be used to control such systems. A few preliminary points have emerged from the small number of papers developed in this area to date. First, for renewable resource models, there is an optimal shadow price of biomass that varies over both time and space (Sanchirico and Wilen 2005). The shadow price at any point in space accounts not only for the contribution of a marginal biomass change to own profits at that point in space, but also to the marginal contribution (via diffusion) to profits in all other patches. This implies that optimal policies must be determined over the entire landscape, or at least over subsets of systems that are linked and connected. The policy questions that are raised by this finding are several. What kinds of command and control policies (eg. taxes, quotas) are capable of resolving the spatial-dynamic externalities? How costly are spatially (and temporally) differentiated instruments to implement and monitor, and what is the efficiency payoff of doing so? How do the costs and benefits depend upon fundamental bioeconomic

parameters and the structural nature of the diffusion system? What kinds of decentralized institutions are feasible and how should they be designed? Is there a role for decentralized cross-patch negotiation to resolve spatial externalities? What kinds of institutional designs would foster spontaneous decentralized arbitrage?

Given the complexity of these spatial-dynamic systems and the challenges of designing policies in such a context, another critical role for empirical work is to parameterize *ex ante* policy analyses of real spatial policy proposals. Even if real-world proposals do not originate with empirical estimates of spatial-dynamic processes, empirical estimates can answer positive questions about how these policies are likely to perform. Consider, for example, the proliferation of proposals to form new marine reserves. Most of these proposals have little connection to underlying spatial economic dynamics or what we might consider a first-best. In an empirically-based *ex ante* spatial-dynamic model, Smith and Wilen (2003) show that conclusions about the fishery effects of marine reserves reverse when one accounts for fishing behavior. This is important information for policy makers, even though Smith and Wilen are not able to specify what the optimal policy would be. Their results are also consistent with theoretical findings of Sanchirico et al (2006) and Costello and Polasky that optimality of reserves hinges on the ability to control effort spatially in other locations.

Sanchirico and Wilen (2005) examine conceptual issues of institutional design by characterizing first- and second-best optimal policies in a simple two patch metapopulation model. They conclude that optimal population densities reflect bioeconomic parameters describing each patch as well as the structural nature of dispersal. In particular, consider a system with dispersal governed by relative densities so that diffusion occurs from high- to low-density patches. If bioeconomic parameters (unit prices, costs, intrinsic growth, carrying

capacities) are identical across all patches, the optimized system will look like a system whose patches have been independently optimized. But a system with bioeconomic heterogeneity will call for integrated system-wide solutions that account for parameter differences as well as the nature of interconnections. In an example with adult migration responsive to relative densities, it is optimal to adjust biomass so that dispersal flows from low profit to high profit patches. The equilibrium is a “flux equilibrium” in which effort and population densities are constant, but which maintains the equilibrium with flux across patch boundaries. The task of management in this spatial-dynamic system is thus one that jointly manages not only steady-state patch-specific profit differences, but also the flows of dispersal between patches in order to maximize system-wide returns.

These results suggest that the conventional “whole fishery” paradigm of managing a fishery as if it is a homogeneous single stock will be inefficient if it is a metapopulation. A metapopulation will require the spatial differentiation of policy instruments in ways that account for spatial externalities. For a fishery, this could mean, for example, patch specific total allowable catches (TACs), or patch specific limited entry licensing programs, or patch specific landings taxes and/or individual quota programs. This kind of instrument proliferation would be costly of course, raising the question: what if the regulator is unable to set policies at the necessary spatially scale for a first-best outcome? Suppose that it is possible to use one a single, undifferentiated instrument, such as a landings tax rather than spatially differentiated landings taxes. How would one optimally set such a tax and what would the implications be? In their two-patch fishery example, Sanchirico and Wilen (2005) show that the inability to implement spatially explicit policies leads the regulatory authority to “over- conserve” the high cost patch and “under- conserve” the low cost patch where the magnitude depends on the degree and nature

of connectivity in the system. Similar conclusions were found in Wilson et al (1999), who also investigate the policy implications of mismatched scales in a fishery context. In general, the results accord with intuition in the sense that optimal second best policies average or blend the spatially differentiated results. With an inability to utilize spatially differentiated instruments, the system cannot be as finely tuned as with first best instruments. As a result, overall fishery rents are lower, total effort is mismatched over space, and the biomass levels are not optimally adjusted to take advantage of the possibility of “farming” the system or aligning the dispersal and economic gradient.

These conceptual descriptions of optimal policies also raise important practical institutional design questions. How do we induce cooperation among autonomous agents and agencies in a system that is linked by spatial-dynamic processes? How do we reward users of patches that generate system-wide externalities to allow those external effects to propagate? A couple of recent papers begin to investigate these issues. For instance, Parkhurst et al. (2002) investigate the potential for using an agglomeration bonus to get landowners to better coordinate land set-asides as a means to conserve contiguous parcels of habitat. While this research is exciting, it also raises many questions about possible uniqueness of Nash equilibria in a spatial context. More recently, Bhat and Huffaker (2007) design self-reinforcing cooperative agreements within a differential game context for two landowners that share a biological resource (beaver population) that disperses between the properties.

Similar questions arise for point-input systems like invasive species problems. How do we induce individuals who are first impacted to account for the spillover benefits of early control of invasions? What kinds of regional institutions are needed to tackle problems such as controlling invasive weed species? What kinds of international institutions are feasible to tackle

phenomena like the avian flu or other potential pandemics? Many of these issues have features like the familiar commons game, but, in addition, they have structures such that the position of particular decision makers in time and space matter for the system outcome. As this frontier area is subjected to more analysis, the answers to these and similar questions will become clearer as economists synthesize principles from conceptual and empirical understanding.

An important aspect of our understanding of spatial systems undoubtedly will come from measuring the effects of spatial policies *ex post*. Our experience with marine reserves and other spatial policies is growing over time, and we will have opportunities to test whether policies to address spatial-dynamic features of coupled human-natural systems work as our *ex ante* models predict. Most evaluation of marine reserves is being done by non-economists who consider spatial policy interventions as if they are randomized experimental trials. However, the treatment and control groups in most of these policy interventions are not randomly assigned because, in coupled systems, humans interact with the resource base (Smith, Zhang, and Coleman 2006). This suggests that program evaluation will also serve an important role for understanding spatial-dynamic systems. Program evaluation is now widely being used for environmental policy, and there are circumstances in which spatial variation or spatial geometry can be used to isolate policy treatment effects (e.g. Greenstone 2004). Spatial policies in renewable resource management will potentially generate the means to identify treatment effects through analyzing outcomes in spatially-explicit longitudinal data sets, whereas evaluating traditional non-spatial fishery management policies must identify treatment effects strictly from time series data. Program evaluation of spatial renewable resource policies will complement empirical work that is used either to design optimal spatial policies or predict the performance of second-best alternatives.

References

- Albers, H.J. 1996. Modeling Ecological Constraints on Tropical Forest Management: Spatial Interdependence, Irreversibility, and Uncertainty, *Journal of Environmental Economics and Management* 30(1), 73-94.
- Ashton, A., A.B. Murray, and O. Arnoult. 2001. Formation of coastline features by large-scale instabilities induced by high-angle waves. *Nature* 414:296-300.
- Bhat, M.G., R.G. Huffaker, and S.M. Lenhart. 1993. Controlling Forest Damage by Dispersive Beaver Populations: Centralized Optimal Management Strategy, *Ecological Applications* 3(3), 518-530.
- Bhat, M.G., R.G. Huffaker, and S.M. Lenhart. 1996. Controlling transboundary wildlife damage: modeling under alternative management scenarios, *Ecological Modeling* 92, 215-224.
- Bhat, M.G. and R.G. Huffaker. 2007. Management of a transboundary wildlife population: A self-enforcing cooperative agreement with renegotiation and variable transfer payments. *Journal of Environmental Economics and Management*. 53:54-67.
- Bockstael, N.E. 1996. Modeling Economics and Ecology: The importance of a spatial perspective, *American Journal of Agricultural Economics* 78 (N5), 1168-1180.
- Brock, William and A. Xepapadeas. 2006. Optimal Control and Spatial Heterogeneity: Pattern Formation in Economic-Ecological Models. Department of Economics, University of Wisconsin, Working Paper.
- Brown, G.M., and Roughgarden, J., 1997. A metapopulation model with private property and a common pool. *Ecological Economics*, 22, 65-71.
- Bulte, E. H. and v.K.G. Cornelis. 1999. Metapopulation dynamics and stochastic bioeconomic modeling, *Ecological Economics* 30, 293-299.
- Clark, C. W. ,1990. *Mathematical Bioeconomics. The Optimal Management of Renewable Resources. Second edition*. John Wiley & Sons, New York: USA.
- Comitini, S. and D.S. Huang. 1967. A study of production and factor shares in the halibut fishing industry. *Journal of Political Economy* 75:366-372.
- Costello, Chris and S. Polasky. 2006. Can Biological Reserves Increase Profits? Optimal Harvesting with Spatial Connectivity. Working Paper, Bren School, UCSB.
- Cowen, R.K., K. M. M. Lwiza, S. Sponaugle, C. B. Paris, and D. B. Olson 2000. Connectivity of Marine Populations: Open or Closed? *Science* 287: 857-859.

- Cowen, R.K. C.B. Paris, and A. Srinivasan. 2006. Scaling Connectivity in Marine Populations. *Science* 311: 522-527.
- De Valpine, P. and A. Hastings. 2002. Fitting population models incorporating process noise and observation error. *Ecological Monographs* 72:57-76.
- Deacon, R.T. 1989. An empirical model of fishery dynamics. *Journal of Environmental Economics and Management* 16:167-83.
- Dupont, D.P. (1993), "Price Uncertainty, Expectations Formation and Fishers' Location Choices," *Marine Resource Economics* 8, 219-247.
- Eales, J. and J.E. Wilen, An examination of fishing location choice in the pink shrimp fishery, *Marine Resource Economics* 2, 331-351 (1986).
- Fisher, RA 1937 The wave of advance of advantageous genes. *Annals of Eugenics*, 7:355-369
- Fujita, M., P. Krugman, and A. Venables. 1999. *The Spatial Economy*. MIT Press. Cambridge, MA.
- Greenstone, M. 2004. Did the Clean Air Act cause the remarkable decline in sulfur dioxide concentrations? *Journal of Environmental Economics and Management* 47:585-611.
- Hastings, A., and L. W. Botsford. 2003. Comparing designs of marine reserves for fisheries and for biodiversity. *Ecological Applications* 13:S65-S71.
- Holland, D.S. and J.G. Sutinen, Location choice in the New England trawl fisheries: old habits die hard, *Land Economics* 76, 133-149 (2000).
- Holland, Daniel S. and Kurt E. Schnier. 2006. Individual Habitat Quotas for Fisheries. *Journal of Environmental Economics and Management*. 51(1):72-92.
- Huffaker, R. G., M. G. Bhat, and S. M. Lenhart. 1992. Optimal trapping strategies for diffusing nuisance-beaver populations. *Natural Resource Modeling* 6:71-97.
- Jannmatt, J. A. 2005. Sharing clams: tragedy of an incomplete commons. *Journal of Environmental Economics and Management* 49:26-51.
- Lenhart, S. and M. Bhat. 1999. Application of Distributed Parameter Control Model in Wildlife Damage Management" *Mathematical Models and Methods in Applied Sciences*, 4: 423-439.
- Liebhold, A.M, J.A. Halvorson, and G.A. Elmes. 1992. Gypsy Moth invasion in Orth America: A quantitative analysis. *Journal of Forestry*, 95: 20-24.

- McFadden, D. 1974. "Conditional Logit Analysis of Qualitative Choice Behavior," in P. Zarembka, ed., *Frontiers in Econometrics*. New York: Academic Press.
- Murray, J.D. 2002. *Mathematical Biology*, vols. I and II, Third Edition, Springer-Verlag, New York.
- Neubert, M. 2003. Marine Reserves and Optimal Harvesting, *Ecology Letters*, 6: 843-849.
- Okubo, A. and S.A. Levin (2002). *Diffusion and Ecological Processes: Modern Perspectives*, Second Edition, Springer-Verlag, New York.
- Parkhurst G.M.; Shogren J.F.¹; Bastian C.; Kivi P.; Donner J.; Smith R.B.W. 2002. Agglomeration bonus: an incentive mechanism to reunite fragmented habitat for biodiversity conservation. *Ecological Economics*, 41(2), May 2002, pp. 305-328(24).
- Petrovskii, S. and B-L Li. 2003. An exactly solvable model of population dynamics with density-dependent migrations and the Allee effect. *Mathematical Biosciences*, 86: 79-91.
- Phaneuf, D.P., J.A. Herriges, and J.C. Carbone. 2007. Non-price equilibria for non-marketed goods. Working Paper. Resource for the Future's Frontiers in Environmental Economics Workshop.
- Polacheck, T., R. Hilborn, and A.E. Punt. 1993. Fitting surplus production models: comparing methods and measuring uncertainty. *Canadian Journal of Fisheries and Aquatic Sciences* 50: 2597-2607.
- Potapov, A, M. Lewis, D. Finnoff. 2006. Optimal Control of Invasions in Lake Networks, *Natural Resource Modeling* (forthcoming).
- Robinson, E., J. Williams, and H.J. Albers. 2002. The Impact of Markets and Policy on Spatial Patterns of Non-Timber Forest Product Extraction. *Land Economics*. 78:2:260-271.
- Sale, P. , Hanski, I., and J. P. Kritzer. 2006. The merging of metapopulation theory and marine ecology: Establishing the historical context. In *Marine Metapopulations* (eds. Kritzer and Sale). Elsevier. New York.
- Sanchirico, J. N. 2004. Designing a Cost-Effective Marine Reserve Network: A Bioeconomic Metapopulation Analysis, *Marine Resource Economics*, 19(1): 46-63.
- Sanchirico, J. N. 2005. Additivity Properties in Metapopulation Models: Implications for the assessment of marine reserves, *J. Environ. Econom. Management* 49(1):1-25.
- Sanchirico, J. N. and J. E. Wilen. 1999. Bioeconomics of spatial exploitation in a patchy environment, *J. Environ. Econom. Management* 37: 129-150.

- Sanchirico, James and James Wilen. 2005. Managing Renewable Resource Use With Market-Based Instruments: Matching Policy Scope to Ecosystem Scale. *Journal of Environmental Economics and Management*, 50:1, 23-46.
- Sanchirico, J.N, Maldavkar, U., A. Hastings, and J. Wilen. 2006. When are no-take zones an economically optimal fishery management strategy? *Ecological Applications*, 16(5):1643-1659.
- Schaefer, M.B. 1954. Some aspects of the dynamics of populations important to the management of commercial marine fisheries. *Bulletin of the Inter-American Tropical Tuna Commission* 1: 26-56.
- Sharov, A. and A. Liebhold. 1998. Bioeconomics of Managing the Spread of Exotic Pests with Barrier Zones, *Ecological Applications*, 8(3): 833-845.
- Skonhofs, A., and Solstad, J.T., 1996. Wildlife Management, Illegal Hunting and Conflicts. A Bioeconomic Analysis. *Environment and Development Economics* 1:165-181.
- Skonhofs, A and C.W. Armstrong. 2006. Marine reserves: A bio-economic model with asymmetric density dependent migration. *Ecological Economics* 57(3): 466-476.
- Skellam, J.G. 1951. Random Dispersal in theoretical populations, *Biometrika* 38, 196-218.
- Smith, M.D. 2002. Two econometric approaches for predicting the spatial behavior of renewable resource harvesters, *Land Economics* 78, 522-538.
- Smith, M.D. 2005. "State Dependence and Heterogeneity in Fishing Location Choice." *Journal of Environmental Economics and Management* 50: 319-340.
- Smith, M.D. and J.E. Wilen. 2002. The marine environment: fencing the last frontier, *Review of Agricultural Economics*, 24(1): 34-42.
- Smith, M.D. and J.E. Wilen. 2003. "Economic Impacts of Marine Reserves: The Importance of Spatial Behavior." *Journal of Environmental Economics and Management* 46: 183-206.
- Smith, M.D. and J.E. Wilen. 2004. "Marine Reserves with Endogenous Ports: Empirical Bioeconomics of the California Sea Urchin Fishery." *Marine Resource Economics* 19: 85-112.
- Smith, M.D. J. Zhang, and F.C. Coleman. 2006. Effectiveness of Marine Reserves for Large-Scale Fisheries Management. *Canadian Journal of Fisheries and Aquatic Sciences* 63:153-164.
- Supriatna, A.K. and H.P. Possingham. 1999. Harvesting a Two-Patch Predator-Prey Metapopulation, *Natural Resource Modeling* 12, 481-198.

- Train, K.E. 2003. *Discrete Choice Methods with Simulation*. Cambridge: Cambridge University Press.
- Tuck, G.N. and Possingham, H.P., 1994. Optimal Harvesting Strategies for a Metapopulation. *Bulletin of Mathematical Biology* 56:107-127.
- Uhler, R.S. 1980. Least squares estimates of the Schaefer production model: some Monte Carlo simulation results. *Canadian Journal of Fisheries and Aquatic Sciences* 37:1284-94.
- Viboud, C., O. Bjornstad, D. Smith, L. Simonsen, M. Miller, and B. Grenfell. 2006. Synchrony, waves, and spatial hierarchies in the spread of influenza, *Science*, 312(5772): 447-451.
- Wilen, J.E. *Common Property Resources and the Dynamics of Overexploitation: The Case of the North Pacific Fur Seal*, University of British Columbia, Resources Paper No. 3, September 1976.
- Wilson, J., B. Low, R. Costanza, and E. Ostrom. 1999. Scale misperceptions and the spatial dynamics of a social-ecological system. *Ecological Economics* 31, 243-257.
- Zhang, J. and M.D. Smith. 2007. Bias correction in an empirical fishery model: a two-stage approach. Working Paper, Duke University.

Table 1- Results of 100 Monte Carlo Simulations to Recover the Structural Spatial-Dynamic Parameters Using Seemingly Unrelated Regression Sink-Source System

$J = 3, n = 100, T = 100, \bar{p} = 100, \sigma_p = 10, \sigma_\varepsilon = 0.075$

Parameter	True Value	Mean	Min	Max	Median	Standard Deviation
<i>Restricted Model - Form of Dispersal Matrix Is Known A Priori</i>						
q	0.015	0.0149	0.0124	0.0166	0.0150	0.0008
d21	0.15	0.1370	0.0239	0.2771	0.1398	0.0485
d31	0.05	0.0453	-0.0891	0.1740	0.0506	0.0573
r1	1.4	1.3755	1.0907	1.6550	1.3813	0.0995
r2	0.5	0.5467	0.1961	0.8646	0.5394	0.1306
r3	0.8	0.8160	0.4994	1.1835	0.8209	0.1409
k1	3	2.9935	2.4892	3.6546	2.9893	0.2082
k2	2	1.9986	1.3754	2.3138	2.0030	0.1414
k3	2	2.0047	1.7430	2.2530	2.0050	0.1143
<i>Unrestricted Model - Form of Dispersal Matrix Not Known A Priori</i>						
q	0.015	0.0149	0.0124	0.0166	0.0150	0.0008
d12	0	-0.0071	-0.1178	0.0833	-0.0012	0.0453
d13	0	0.0101	-0.0865	0.1128	0.0046	0.0451
d21	0.15	0.1360	0.0029	0.2874	0.1375	0.0516
d23	0	0.0049	-0.1182	0.1239	0.0044	0.0505
d31	0.05	0.0433	-0.1053	0.1939	0.0451	0.0588
d32	0	0.0094	-0.0962	0.1114	0.0092	0.0430
r1	1.4	1.3688	1.0324	1.6196	1.3689	0.1089
r2	0.5	0.5446	0.1303	0.8978	0.5591	0.1522
r3	0.8	0.8236	0.5004	1.2395	0.8149	0.1563
k1	3	2.9811	2.4832	3.7387	2.9893	0.2152
k2	2	1.9899	0.9033	2.8532	1.9905	0.3411
k3	2	2.0295	1.5785	2.9698	2.0286	0.2199

Table 2- Results of 100 Monte Carlo Simulations to Recover the Structural Economic Parameters in Second Stage

Parameter	True Value	Mean	Median	Min	Max	Standard Deviation
<i>Raw Estimates</i>						
$\alpha + c$	1.000	0.5211	0.5153	0.2823	0.9824	0.1165
ϕ	0.500	0.2491	0.2485	0.2263	0.2715	0.0090
Revenue	1	0.5095	0.5047	0.4129	0.7006	0.0519
<i>Transformed by Estimated Scale - True Scale = 2.0</i>						
$\alpha + c$	1.000	1.0115	0.9990	0.6837	1.4022	0.1252
ϕ	0.500	0.4944	0.4948	0.3274	0.6433	0.0562

Table 3 - Results of 100 Monte Carlo Simulations to Recover the Structural Spatial-Dynamic Parameters Using Seemingly Unrelated Regression Density-Dependent System

$J = 3, n = 100, T = 100, \bar{p} = 100, \sigma_p = 10, \sigma_\varepsilon = 0.15$

Parameter	True Value	Mean	Median	Min	Max	Standard Deviation
<i>Overidentified Density-Dependent Model</i>						
q	0.015	0.0149	0.0149	0.0133	0.0160	0.0006
d11	-0.250	-0.2380	-0.2364	-0.3828	-0.1039	0.0605
d12	0.083	0.0850	0.0866	-0.0487	0.1919	0.0472
d13	0.167	0.1690	0.1745	0.0714	0.2559	0.0404
d21	0.083	0.0771	0.0805	-0.0424	0.1786	0.0475
d22	-0.333	-0.3372	-0.3449	-0.4425	-0.1135	0.0661
d23	0.250	0.2438	0.2473	0.1206	0.3854	0.0480
d31	0.167	0.1609	0.1648	0.0414	0.2555	0.0488
d32	0.250	0.2522	0.2550	0.1302	0.3644	0.0478
d33	-0.417	-0.4127	-0.4104	-0.5947	-0.2390	0.0625
r1	1.000	0.9748	0.9471	0.5735	1.6273	0.1933
r2	1.000	1.0359	1.0473	0.4571	1.5451	0.1917
r3	1.000	1.0030	1.0034	0.6053	1.3843	0.1764
k1	3.000	2.9856	3.0147	2.4304	3.4663	0.2267
k2	3.000	3.0225	3.0523	2.4211	3.5426	0.2155
k3	3.000	2.9991	2.9926	2.3346	3.5571	0.2174

Table 4 - Results of 100 Monte Carlo Simulations to Recover the Structural Spatial-Dynamic Parameters Using Seemingly Unrelated Regression Density-Dependent System With Observation Error

$J = 3, n = 100, T = 100, \bar{p} = 100, \sigma_p = 10, \sigma_\varepsilon = 0.15$

Parameter	True Value	Mean	Median	Min	Max	Standard Deviation	Not a Number*
Overidentified Density-Dependent Model - Observation Error Sig=0.1							
q	0.0150	0.0149	0.0149	0.0125	0.0172	0.001	0
d11	-0.2500	-0.2215	-0.2229	-0.5286	0.1075	0.082	0
d12	0.0833	0.0725	0.0728	-0.1332	0.2588	0.051	0
d13	0.1667	0.1492	0.1502	-0.0763	0.3360	0.050	0
d21	0.0833	0.0701	0.0689	-0.1373	0.2725	0.055	0
d22	-0.3333	-0.3015	-0.3010	-0.5594	0.0107	0.076	0
d23	0.2500	0.2275	0.2278	0.0700	0.4101	0.049	0
d31	0.1667	0.1514	0.1522	-0.0792	0.3720	0.059	0
d32	0.2500	0.2290	0.2285	0.0229	0.4117	0.053	0
d33	-0.4167	-0.3768	-0.3769	-0.6591	-0.1183	0.072	0
r1	1.0000	1.0669	1.0674	0.2314	1.8718	0.215	0
r2	1.0000	1.0717	1.0690	0.3119	2.0024	0.201	0
r3	1.0000	1.0556	1.0575	0.3460	1.8596	0.201	0
k1	3	2.8942	2.8875	1.5036	4.1394	0.279	0
k2	3	2.9116	2.9101	1.5285	3.9205	0.255	0
k3	3	2.9058	2.9027	1.7163	3.8868	0.254	0
Overidentified Density-Dependent Model - Observation Error Sig=1.0							
q	0.015	0.0153	0.0153	-0.0037	0.0303	0.0044	1
d11	-0.250	-0.4309	-0.4194	-1.2442	0.1768	0.2015	1
d12	0.083	0.1350	0.1315	-0.1641	0.5095	0.0935	1
d13	0.167	0.1384	0.1344	-0.1314	0.5331	0.0853	1
d21	0.083	0.1872	0.1813	-0.1980	0.7879	0.1239	1
d22	-0.333	-0.3725	-0.3637	-1.1123	0.1563	0.1674	1
d23	0.250	0.1872	0.1815	-0.1210	0.6542	0.1011	1
d31	0.167	0.2433	0.2363	-0.2073	0.9236	0.1380	1
d32	0.250	0.2371	0.2308	-0.2282	0.7633	0.1232	1
d33	-0.417	-0.3260	-0.3193	-0.9288	0.1115	0.1431	1
r1	1.000	1.3427	1.3745	-0.4952	2.8021	0.4548	1
r2	1.000	1.1518	1.1821	-0.7502	2.7089	0.4609	1
r3	1.000	0.9287	0.9565	-0.8888	2.1696	0.4403	1
k1	3.000	2.9387	3.1285	-1605	61.2	23.4	1
k2	3.000	2.8915	3.1214	-1557.2	305.5	22.9	1
k3	3.000	2.7209	2.9505	-373.164	398.2688	11.3839	1
Overidentified Density-Dependent Model - Observation Error Sig=2.0							
q	0.015	-0.0142	-0.0128	-0.3298	0.3686	0.0307	2
d11	-0.250	-0.4874	-0.3686	-12.9063	1.9023	0.6196	2
d12	0.083	0.1589	0.1037	-1.9886	7.5357	0.2772	2
d13	0.167	0.1281	0.0842	-3.5595	6.5235	0.2586	2
d21	0.083	0.2347	0.1601	-3.3590	9.7365	0.4143	2
d22	-0.333	-0.3597	-0.2644	-9.0540	7.7592	0.4699	2
d23	0.250	0.1478	0.0999	-4.3462	5.3882	0.2809	2
d31	0.167	0.2527	0.1757	-1.6992	11.8684	0.4032	2
d32	0.250	0.2008	0.1381	-8.4907	7.6496	0.3406	2
d33	-0.417	-0.2759	-0.2045	-6.8424	4.1885	0.4039	2
r1	1.000	0.0072	-0.0540	-6.7300	15.3708	1.1412	2
r2	1.000	-0.1438	-0.1668	-6.1892	11.0015	0.9329	2
r3	1.000	-0.2581	-0.2782	-5.6480	7.7297	0.8213	2
k1	3.000	25.7282	9.1508	-52890	105430	2390	2
k2	3.000	-16.6114	10.9632	-487770	107030	7540	2
k3	3.000	-483.906	12.0781	-398980	128510	6070	2

* Number of occurrences out of 5,000 with a divide by zero error or other error in the estimation.

Appendix A – Recovering Structural Bio-Physical Parameters in 3-Patch System

When adding up holds, we can recover all of the structural bio-physical parameters of the spatial-dynamic system from the reduced-form parameters. In the 3-patch system, there are thirteen total parameters.

$$q = -\beta_{15}$$

$$\begin{aligned} d_{11} &= -\beta_{23} - \beta_{33} & d_{21} &= \beta_{23} & d_{31} &= \beta_{33} \\ d_{12} &= \beta_{13} & d_{22} &= -\beta_{13} - \beta_{34} & d_{32} &= \beta_{34} \\ d_{13} &= \beta_{14} & d_{23} &= \beta_{24} & d_{33} &= -\beta_{14} - \beta_{24} \end{aligned}$$

$$r_1 = \beta_{11} - 1 + d_{21} + d_{31} \quad r_2 = \beta_{21} - 1 + d_{12} + d_{32} \quad r_3 = \beta_{31} - 1 + d_{13} + d_{23}$$

$$K_1 = \frac{-r_1}{q\beta_{12}} \quad K_2 = \frac{-r_2}{q\beta_{22}} \quad K_3 = \frac{-r_3}{q\beta_{32}}$$

Appendix B – Additional Experiments

**Table B-1- Results of 100 Monte Carlo Simulations to Recover the Structural Spatial-Dynamic Parameters Using Seemingly Unrelated Regression
Increased Process Error**

$J = 3, n = 100, T = 100, \bar{p} = 100, \sigma_p = 10, \sigma_\varepsilon = 0.3$

Parameter	True Value	Mean	Min	Max	Median	Standard Deviation
<i>Restricted Model - Form of Dispersal Matrix Is Known A Priori</i>						
q	0.015	0.0148	0.0071	0.0210	0.0149	0.0024
d21	0.15	0.1549	-0.0486	0.3172	0.1545	0.0777
d31	0.05	0.0387	-0.1192	0.2001	0.0376	0.0804
r1	1.4	1.3724	0.9752	1.8141	1.3855	0.1747
r2	0.5	0.5213	0.0981	0.9915	0.5074	0.2037
r3	0.8	0.8490	0.1103	1.4017	0.8477	0.2297
k1	3	3.1719	2.2285	5.5090	2.9961	0.5851
k2	2	1.9411	1.1134	3.3591	1.9378	0.3388
k3	2	2.0382	1.6438	3.0297	2.0135	0.2382
<i>Unrestricted Model - Form of Dispersal Matrix Not Known A Priori</i>						
q	0.015	0.0148	0.0071	0.0216	0.0147	0.0028
d12	0	0.0051	-0.2275	0.1809	0.0100	0.0756
d13	0	-0.0033	-0.1428	0.1599	-0.0029	0.0610
d21	0.15	0.1499	-0.0750	0.3449	0.1485	0.0839
d23	0	0.0140	-0.1321	0.1830	0.0145	0.0659
d31	0.05	0.0406	-0.1329	0.2234	0.0461	0.0876
d32	0	-0.0008	-0.1568	0.2764	-0.0037	0.0690
r1	1.4	1.3696	0.8398	1.7293	1.3782	0.2021
r2	0.5	0.5119	-0.1311	1.1734	0.5120	0.2263
r3	0.8	0.8552	0.0564	1.4827	0.8117	0.2645
k1	3	3.1719	2.2212	5.2493	2.9790	0.5951
k2	2	1.9034	-5.2289	4.9334	1.9727	0.9686
k3	2	2.0844	0.8823	4.1149	2.0019	0.4933

**Table B-2 - Results of 100 Monte Carlo Simulations to Recover the Structural Economic Parameters in Second Stage
Increased Process Error**

$J = 3, n = 100, T = 100, \bar{p} = 100, \sigma_p = 10, \sigma_\varepsilon = 0.3$

Parameter	True Value	Mean	Median	Min	Max	Standard Deviation
Raw Estimates						
$\alpha + c$	1.000	0.5211	0.5153	0.2823	0.9824	0.1165
ϕ	0.500	0.2491	0.2485	0.2263	0.2715	0.0090
Revenue	1	0.5095	0.5047	0.4129	0.7006	0.0519
Transformed by Estimated Scale - True Scale = 2.0						
$\alpha + c$	1.000	0.9996	0.9981	0.7372	1.1481	0.0704
ϕ	0.500	0.5008	0.4967	0.3972	0.6602	0.0437

Table B-3- Results of 100 Monte Carlo Simulations to Recover the Structural Spatial-Dynamic Parameters Using Seemingly Unrelated Regression
Fix r and K across patches

$J = 3, n = 100, T = 100, \bar{p} = 100, \sigma_p = 10, \sigma_\varepsilon = 0.15$

Parameter	True Value	Mean	Min	Max	Median	Standard Deviation
<i>Restricted Model - Form of Dispersal Matrix Is Known A Priori</i>						
q	0.015	0.0151	0.0090	0.0207	0.0151	0.0018
d21	0.15	0.1643	-0.1232	0.3690	0.1653	0.0843
d31	0.05	0.0397	-0.1563	0.2126	0.0387	0.0750
r1	0.8	0.8160	0.5247	1.1606	0.8292	0.1391
r2	0.8	0.7978	0.5183	1.0674	0.7924	0.1216
r3	0.8	0.8318	0.5434	1.2003	0.8168	0.1170
k1	2	2.1094	0.9652	5.8733	1.9296	0.7162
k2	2	1.9961	1.5963	2.7447	1.9769	0.1669
k3	2	1.9939	1.6107	2.5536	1.9663	0.1610
<i>Unrestricted Model - Form of Dispersal Matrix Not Known A Priori</i>						
q	0.015	0.0151	0.0096	0.0205	0.0151	0.0018
d12	0	0.0014	-0.1347	0.1400	0.0016	0.0569
d13	0	0.0057	-0.1421	0.1576	0.0038	0.0604
d21	0.15	0.1683	-0.1196	0.4054	0.1765	0.0883
d23	0	-0.0112	-0.1415	0.1542	-0.0153	0.0608
d31	0.05	0.0430	-0.1876	0.2428	0.0448	0.0801
d32	0	-0.0007	-0.1883	0.1491	-0.0026	0.0647
r1	0.8	0.8086	0.4944	1.2848	0.8091	0.1639
r2	0.8	0.8104	0.4527	1.1887	0.8281	0.1502
r3	0.8	0.8256	0.3595	1.1791	0.8327	0.1606
k1	2	2.1211	0.9737	5.9289	1.9667	0.7099
k2	2	2.0156	1.3369	3.2140	1.9766	0.3178
k3	2	1.9721	1.3677	2.8348	1.9422	0.2906